Solutions of Permutation and Combination | WB - 2

Miscellaneous	**Level – 1**

1.(C) We have: 1.1! + 2.2! + 3.3! + ... + $n.n!$

$$= \sum_{r=1}^{n} r\,.\,(r!) = \sum_{r=1}^{n} (r+1)r! - r! = \sum_{r=1}^{n}\left[(r+1)! - r!\right]$$

$$= (2!-1!) + (3!-2!) + \ldots + \left[(n+1)! - n!\right] = (n+1)! - 1! = (n+1)! - 1$$

2.(B) Use 20! = $2^{18}\ 3^{8}\ 5^{4}\ 7^{2}$ (11) (13) (17) (19)

3.(B) $x_1 + x_2 + x_3 = 8,\ 1 \le x_1, x_2, x_3 \le 6$

$\Rightarrow$ The required answer is the co-efficient of x^8 in $\left(x + x^2 + \ldots + x^6\right)^3$

Or the co-efficient of x^5 in $\left(1 + x + \ldots + x^5\right)^3 = \left(1 - x^6\right)^3 (1-x)^{-3}$

$\Rightarrow$ The required answer is ${}^{7}C_5 = 21$

4.(D) Required number of possible outcomes

= Total number of possible outcomes – Number of possible outcomes in which 5 does not appear on any dice $= 6^3 - 5^3 = 216 - 125 = 91$

5.(D) Total combinations possible: $(2, 2, 2) \to 1$ 3 digit number

$(1, 1, 1) \to 1$ 3 digit number

$(2, 1, 0) \to (3! - 2!) = 4$ 3-digit number

Total numbers = 6

6.(D) The sum of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 is 45. So, basically, we have to remove 2 digits from the above list such that their sum is 9 because only in those cases we will get an 8-digit number divisible by 9.

When we remove 0 and 9. $\underline{|8}$ numbers can be formed. On removing any other pair with sum 9 (4 such pairs), we have $7 \times \underline{|7}$ numbers. So, total numbers = $\underline{|8} + \left(4 \times 7 \times \underline{|7}\right) = 36\left(\underline{|7}\right)$

7.(B) There are 9 women and 8 men. A committee of 12, consisting of at least 5 women, can be formed by choosing:

(i) 5 women and 7 men **(ii)** 6 women and 6 men

(iii) 7 women and 5 men **(iv)** 8 women and 4 men

(v) 9 women and 3 men

$\therefore$ Total number of ways of forming the committee

$= {}^{9}C_5 \times {}^{8}C_7 + {}^{9}C_6 \times {}^{8}C_6 + {}^{9}C_7 \times {}^{8}C_5 + {}^{9}C_8 \times {}^{8}C_4 + {}^{9}C_9 \times {}^{8}C_3$

$= 126 \times 8 + 84 \times 28 + 36 \times 56 + 9 \times 70 + 1 \times 56 = 6062$

8.(B) 18 correct results can be predicted in ${}^{21}C_{18}$ ways and 3 wrong results in 2^3 ways. Thus, required number of ways is ${}^{21}C_{18}\,2^3$.

9.(C) We have 32 places for teeth. For each place we have two choices either there is a tooth or there is no tooth. Therefore, the number of ways to fill up these places is 2^{32}. As there is no person without a tooth, the maximum population is $2^{32}-1$.

10.(A) Number of ways $= {}^5C_1 + {}^5C_2 + {}^5C_3 = 25$

11.(B) Number of ways is ${}^{12}C_6$ for M, ${}^6C_4$ for P and ${}^2C_2$ for C.

Thus, the required number of ways $= \left({}^{12}C_6\right)\left({}^6C_4\right)\left({}^2C_2\right)$

12.(A) We can choose one denomination in ${}^{13}C_1$ ways, then 3 cards of this denomination can be chosen in ${}^4C_3$ ways and one remaining card can be chosen in ${}^{48}C_1$ ways. Thus, the total number of choices is $\left({}^{13}C_1\right)\left({}^4C_3\right)\left({}^{48}C_1\right) = 13 \times 4 \times 48 = 2496$

13.(A) Let x_1, x_2, x_3, x_4 be the number of times T, I, D, E appears on the coupon. Then we must have $x_1 + x_2 + x_3 + x_4 = 8$, where $1 \le x_1, x_2, x_3, x_4 \le 8$ (as each letter must appear once). Then the required number of combinations of coupons is equivalent to the number of positive integral solutions of the above equation, which is further equivalent to number of ways of 8 identical objects distributed among 4 persons when each gets at least one object, and is given by ${}^{8-1}C_{4-1} = {}^7C_3 - 1$. (the solution $x_1 = x_2 = x_3 = x_4 = 2$ is not allowed).

14.(A) 9 diff. balls in 3 diff. boxes

	B-1	B-2	B-3	3 ways
A	5	2	2	Section 6.3
B	4	3	2	Section 6.1
C	3	3	3	Section 6.2

$$A \to \frac{9!}{5!2!2!} \times \frac{1}{2!} \times 3! \qquad B \to \frac{9!}{4!3!2!} \times 3! \qquad C \to \frac{9!}{3!3!3!}$$

$$\text{Total ways} = 9!\left[\frac{3!}{5!(2!)^3} + \frac{3!}{4!3!2!} + \frac{1}{(3!)^3}\right] = 11508$$

15.(C) If we want to fill AAABBB, so that none of R_1, R_2, R_3, R_4, R_5 remains empty then we are to use R_2 and R_4. From R_1, R_3 and R_5 use any two boxes of one of the three rows and one box of remaining two rows.

$\therefore$ Number of ways of selection $= \underset{R_2}{1} \times \underset{R_4}{1} \left(\underbrace{3\left({}^3C_2\right)}_{R_1 \text{ or } R_3 \text{ or } R_5 \text{ uses 2 boxes}} \times \underbrace{{}^3C_1}_{\text{single box}} \times \underbrace{{}^3C_1}_{\text{single box}} \right)$

$= 3 \times (3) \times 3 \times 3 = 81$

$\therefore$ Required ways of permutations of AAABBB in these 81 ways of selection

$= \dfrac{81 \times 6!}{3! \times 3!} = \dfrac{81 \times 720}{6 \times 6} = 1620$

16.(C) Reducing the equation to a newer equation, where sum of variables is less. Thus, finding the number of arrangements becomes easier.

As, $n_1 \geq 1, n_2 \geq 2, n_3 \geq 3, n_4 \geq 4, n_5 \geq 5$

Let $n_1 - 1 = x_1 \geq 0, n_2 - 2 = x_2 \geq 0, ..., n_5 - 5 = x_5 \geq 0$

$\Rightarrow$ New equation will be $x_1 + 1 + x_2 + 2 + ... + x_5 + 5 = 20$

$\Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 = 20 - 15 = 5$

Now, $x_1 \leq x_2 \leq x_3 \leq x_4 \leq x_5$

x_1	x_2	x_3	x_4	x_5
0	0	0	0	5
0	0	0	1	4
0	0	0	2	3
0	0	1	1	3
0	0	1	2	2
0	1	1	1	2
1	1	1	1	1

So, 7 possible cases will be there.

17.(C) For every object we have 2 choices: either put it in the subset or not.

$\Rightarrow$ The total number of subset = $2 \times 2 \times 2 \times . . . .$ n times = 2^n

$2^n = {}^nC_0 + {}^nC_1 + ... + {}^nC_n$

18.(B) At the first place from left we can put only 1. At every other place we can put either 0 or 1.

So, the total of numbers $= 1 \times 2 \times 2 \times 2 \times . . . .9$ times = 2^9.

19.(A) ${}^nC_2 = 36 \quad \Rightarrow \quad n = 9$

20.(B) $\dfrac{\lfloor 2n+1}{\lfloor n+2} \times \dfrac{\lfloor n-1}{\lfloor 2n-1} = \dfrac{3}{5} \quad \Rightarrow \quad \dfrac{(2n+1)(2n)}{(n+2)(n+1)(n)} = \dfrac{3}{5} \quad \Rightarrow \quad n = 4$

21.(A) Required number of ways $= \dfrac{\lfloor 9}{(\lfloor 3)^4} = 280$

22.(D) Required answer = Total no. of numbers – No. of numbers in which 2 is at extreme left

$= \dfrac{\lfloor 5}{\lfloor 2} - \dfrac{\lfloor 4}{\lfloor 2} = 60 - 12 = 48$

23.(C) Total no. of subsets = 2^9 = 512

No. of ways when selecting only one even number = ${}^4C_1 = 4$

No. of ways when selecting only two even numbers = ${}^4C_2 = 6$

No. of ways when selecting only three even numbers = ${}^4C_3 = 4$

No. of ways when selecting only four even numbers = ${}^4C_4 = 1$

Required number of ways = $512 - (4 + 6 + 4 + 1) - 1 = 496$

(We subtract 1 due to the null set)

24.(D) $^{16}C_r = {}^{16}C_{r+2} \Rightarrow r = 7 \quad \Rightarrow \quad {}^{r}P_{r-3} = {}^{7}P_4 = 840$

25.(C) Case 1: Five 1's, one 2, one 3 No. of numbers = $\dfrac{\lfloor 7}{\lfloor 5} = 42$

Case 2: Four 1's, three 2's No. of numbers = $\dfrac{\lfloor 7}{\lfloor 4 \lfloor 3} = 35$

26.(D) Given 10 identical white balls, 9 identical green balls and 7 black balls.

The number of ways selecting atleast one ball.

Number of ways to choose zero or more white balls = (10 + 1)

Number of ways to choose zero or more green balls = (9 + 1)

Number of ways to choose zero or more black balls = (7 + 1)

Hence, number of ways to choose zero or more balls of any colour = (10 + 1) (9 + 1) (7 + 1)

Also, number of ways to choose a total of zero balls = 1

Hence, the number, if ways to choose atleast one ball (irrespective of any colour)

= (10 + 1) (9 + 1) (7 + 1) – 1 = 879

27.(A) Let the number of ways of distributing n identical objects among r persons such that each person gets atleast one object is same as the number of ways of selecting $(r-1)$ places out of $(n-1)$ different places, i.e. $^{n-1}C_{r-1}$]

Statement 1: Here, n = 10 and r = 4

$\therefore$ Required number of ways = $^{10-1}C_{4-1} = {}^{9}C_3$

Statement 2: Required number of ways = $^{9}C_3$

Hence, both the statements are true, but Statement 2 is not a correct explanation of Statement 1.

28.(B) If out of n points, m is collinear, then

Number of triangles = $^{n}C_3 - {}^{m}C_3$

$\therefore$ Number of triangles = $^{10}C_3 - {}^{6}C_3 = 120 - 20 \Rightarrow$ N = 100

29.(C) The number of ways in which 4 novels can be selected $= {}^{6}C_4 = 15$

The number of ways in which 1 dictionary can be selected $= {}^{3}C_1 = 3$

Now, we have 5 places in which middle place is fixed.

$\therefore$ 4 novels can be arrangement in 4! ways.

$\therefore$ The total number of ways = 15 × 24 × 3 = 15 × 24 × 3 = 1080

30.(D) Since, the number of ways that child can buy the six ice-creams is equal to the number of different ways of arranging 6 A's and 4 B's in a row. So, number of ways to arrange 6 A's and 4 B's in a row $= \dfrac{10!}{6!4!} = {}^{10}C_4$

And number of integral solutions of the equation

$x_1 + x_2 + x_3 + x_4 + x_5 = 6 = {}^{6+5}C_{5-1} = {}^{15}C_4 \neq {}^{15}C_5$

Hence, Statement 1 is false, and Statement 2 is true.

31.(B) Total number of ways $= {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$ = 10 + 45 + 120 + 210 = 385

32.(A) Given that, $f(x) = {}^{7-x}P_{x-3}$. The above function is defined, if $7-x \geq 0$ and $x-3 \geq 0$ and $7-x \geq x-3$.

$\Rightarrow \quad x \leq 7, x \geq 3$ and $x \leq 5$

$\therefore \quad D_f = \{3, 4, 5\}$

Now, $\quad f(3) = {}^4P_0 = 1$; $f(4) = {}^3P_1 = 3$ and $f(5) = {}^2P_2 = 2 \quad \therefore \quad R_f = \{1, 2, 3\}$

33.(12) The total number of ways in which 5 men can sit around a table = 4!.
The total number of ways in which two particular people can sit together is 3! 2!.
Therefore, total number of ways in which they don't sit together = 4! – 3!2! = 12.

34.(D) Total number of ways $= \left({}^{10}C_3 \times 2\right) \times 7! = \dfrac{10!}{3}$

35.(A) Same as **122** with $m = n = 5$

36.(ABC) A n-sided polygon has n angular points. Number of triangles formed from these n angular points = ${}^nC_3$.

But it also includes the triangles with sides on the polygon.

Let us consider a side PQ. If each angular point of the remaining (n – 2) points is joined with PQ, we get a triangle with one side PQ.

$\therefore$ Number of triangles with PQ as one side

= n –2 In similar ways n sides like QR can be considered. Hence number of triangles

= n (n –2).

But some triangles have been counted twice. For example, PQ side with R gives ΔPQR and QR side with P gives same ΔPQR.

Number of such triangles = n

[$\therefore$ For each side, one triangle is repeated. Hence for n sides, $n\Delta$ s has been counted more.]

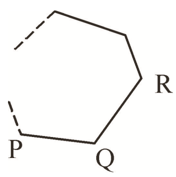
Hence, the number of triangles of which one side is the side of the triangle.

= n (n – 2) – n = n (n–3)

Hence number of required triangles

$= {}^nC_3 - n(n-3)$

$= \dfrac{n(n-1)(n-2)}{6} - n(n-3)$



$= \dfrac{n}{6}(n^2 - 9n + 20)$

$= \dfrac{n}{6}(n-4)(n-5)$

Also, number of ways = Number of positive integral solutions of

$\left[\sum_{i=1}^{3} x_i = n-3\right] \times \dfrac{{}^nC_1}{3} = \dfrac{n}{3} \times {}^{n-4}C_2$

Or Alternatively if we consider linear arrangements

Number of solutions of $\sum_{i=1}^{4} x_i = n-3$ $[x_1, x_4 \geq 0$ and $x_2, x_3 \geq 1]$ minus

Number of solutions with $x_1 = x_4 = 0$

$= {}^{n-2}C_3 - {}^{n-4}C_1$

37.(6) Say the total number of jokes with him is "n", then the total number of triples that can be made out of n is ${}^nC_3$.

The least value of n that satisfies ${}^nC_3 \geq 12$ is 6.

38.(AC) Let person P_i get x_i number of things such that $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 30$

If x_i is odd or $x_i = 2\lambda_i + 1$, where $\lambda_i \geq 0$, then $2(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6) + 6 = 30$

or $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 = 12$

Then number of solutions is ${}^{12+6-1}C_{6-1} = {}^{17}C_5$. if x_i is even or $x_i = 2\lambda_I$, where $\lambda_i \geq 1$, then

$2(\lambda_i + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_6) = 30$ or $\lambda_i + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 = 15$

Therefore, required number of ways is ${}^{15-1}C_{6-1} = {}^{14}C_5$.

39.(AC) Let $x_i\ (1 \leq i \leq n)$ be the number of objects selected of the i^{th} type. Since each object is to be selected at least once, we must have $x_i \geq 1$ and $x_1 + x_2 + ... + x_n = r$. We have to find the number of positive integral solutions of the above equation. Total number of such solutions is ${}^{r-1}C_{n-1} = {}^{r-1}C_{r-n}$.

40.(AD) Problem is same as dividing 17 identical things in two groups.

$\therefore \quad n = \frac{17+1}{2} = 9$

There is no effect if two diamonds are different as necklace can be flipped over. Hence, $n = m = 9$.

41.(BC) Let brothers be denoted by B_1, B_2. First, we need to select the 2 students to be seated between the brothers in ${}^8C_2$ ways.

$\boxed{B_1\ S_1\ S_2\ B_2}$ $\quad \overline{\ 6\ }$

Now the 4 behave like a group. There are $6+1=7$ groups which are arranged in a circle in $\underline{|7-1} = \underline{|6}$ ways. Further the 2 students and 2 brothers can be arranged among themselves in $\underline{|2} \times \underline{|2}$ ways.

So, total by FPC = ${}^8C_2 \times \underline{|6} \times \underline{|2} \times \underline{|2} = {}^8P_2 \times \underline{|6} \times \underline{|2}$

42.(AB) 5 men are arranged around a circular table in $\underline{|5-1} = \underline{|4}$ ways. There are 5 places around them of which we need 4 in ${}^5C_4 = 5$ ways. Then the 4 women are arranged in $\underline{|4}$ ways.

So total by FPC = $\underline{|4} \times 5 \times \underline{|4} = (\underline{|4})^2 \times 5 = \underline{|5} \times \underline{|4}$

43.(CD) Considering the 3 persons as 1 group, we have $7+1=8$ groups which are arranged in a circle in $\lfloor 8-1 = \lfloor 7$ ways.

Then the 3 persons can be arranged among themselves in $\lfloor 3$ ways.

So, by FPC, total $= \lfloor 7 \times \lfloor 3 = \lfloor 6 \times 7 \times \lfloor 3 = \lfloor 6 \times 42$

44.(AC) $^{n+5}P_{n+1} = \dfrac{11(n-1)}{2} \times {}^{n+3}P_n$

or $^{n+5}P_{n+1} = \dfrac{(n+5)!}{4!} = \dfrac{11(n-1)}{2}\dfrac{(n+3)!}{3!}$ or $(n+5)(n+4) = 22(n-1)$

After solving, we get $n = 6$ or $n = 7$.

The number of points of intersection of lines is $^6C_2$ or $^7C_2 = 15$ or 21.

45.(AC) $\because$ $^nC_4, {}^nC_5, {}^nC_6$ are in A.P. $\Rightarrow$ $\dfrac{(n)(n-1)(n-2)(n-3)}{1.2.3.4}, \dfrac{n(n-1)(n-2)(n-3)(n-4)}{1.2.3.4.5},$

$\dfrac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{1.2.3.4.5.6}$ are in A.P.

Dividing each by $\dfrac{n(n-1)(n-2)(n-3)}{1.2.3.4}$ $(\because n \neq 0,1,2,3)$

$\therefore$ $1, \dfrac{n-4}{5}, \dfrac{(n-4)(n-5)}{5.6}$ are in A.P.

$\Rightarrow$ $\dfrac{2(n-4)}{5} = 1 + \dfrac{(n-4)(n-5)}{5.6} \Rightarrow 12n - 48 = 30 + n^2 - 9n + 20$

$\Rightarrow$ $n^2 - 21n + 98 = 0 \Rightarrow (n-7)(n-14) = 0 \Rightarrow n = 7,\ 14$

46.(AC) $\dfrac{\lfloor n+5}{\lfloor (n+5)-(n+1)} = \dfrac{11(n-1)}{2}.\dfrac{\lfloor n+3}{\lfloor (n+3)-n}$ $\Rightarrow$ $\dfrac{\lfloor n+5}{\lfloor 4} = \dfrac{11(n-1)}{2}.\dfrac{\lfloor n+3}{\lfloor 3}$

$\Rightarrow$ $\dfrac{(n+5)(n+4)}{4} = \dfrac{11(n-1)}{2}$

$\Rightarrow n^2 + 9n + 20 = 22n - 22 \Rightarrow n^2 - 13n + 42 = 0 \Rightarrow (n-7)(n-6) = 0 \Rightarrow n = 6$ or $n = 7$

47.(BD) Exponent of 2 in 100! $= \left[\dfrac{100}{2}\right] + \left[\dfrac{100}{4}\right] + \left[\dfrac{100}{8}\right] + \left[\dfrac{100}{16}\right] + \left[\dfrac{100}{32}\right] + \left[\dfrac{100}{64}\right]$

$= 50 + 25 + 12 + 6 + 3 + 1 = 97$ where [] denotes greatest integer function. Exponent of 3 in

$100! = \left[\dfrac{100}{3}\right] + \left[\dfrac{100}{9}\right] + \left[\dfrac{100}{27}\right] + \left[\dfrac{100}{81}\right] = 33 + 11 + 3 + 1 = 48$.

Exponent of 2 = 97 $\Rightarrow$ Exponent of $2^2 = 48$ $\left[2^{97} = \left(2^2\right)^{48}.2\right]$

$\therefore$ 12 will be formed 48 times $\left[\text{using } 2^2 \text{ and } 3\right]$

$^{8}P_{2} = \frac{\lfloor 8}{\lfloor 6} = 7 \times 8 = 56$, $^{8}P_{2} - 8 = 56 - 8 = 48$

48.(C) There are two possible cases

Case I: Six 1's, one 3

Number $= \frac{7!}{6!} = 7$

Case II five 1's, two 2's

Number $= \frac{7!}{5!2!} = 21$ $\therefore$ Total number $= 7 + 21 = 28$

49.(A) If ENDEA is fixed word, then assume this as a single letter.

Total number of letters $= 5$

Total number of arrangements $= 5!$.

50.(D) If E is at first and last places, then total number of permutations $= 7!/2! = 21 \times 5!$

51.(B) If D, L, N are not in last five positions $\leftarrow D, L\ N,\ N \rightarrow \leftarrow E, E,\ E,\ A, O \rightarrow$

Total number of permutations $= \frac{4!}{2!} \times \frac{5!}{3!} = 2 \times 5!$

52.(B) Total number of odd positions $= 5$

Permutations of AEEEO are $\frac{5!}{3!}$.

Total number of even positions $= 4$

$\therefore$ Number of permutations of N, N, D, L $= \frac{4!}{2!}$

$\Rightarrow$ Total number of permutations $= \frac{5!}{3!} \times \frac{4!}{2!} = 2 \times 5!$

53.(D) We have to select 4 seats for 4 persons so that no two persons are together. It means that there should be atleast one empty seat vacant between any two persons.

To place 4 persons, we have to select 4 seats between the remaining 8 empty seats so that all persons should be separated.

Between 8 empty seats 9 seats are available for 4 person to sit.

Select 4 seats in $^{9}C_{4}$ ways.

But we can arrange 4 persons on these 4 seats in 4 ! ways. So total number of ways to give seats to 4 persons so that no two of them are together = $^{9}C_{4} \times 4! = {}^{9}P_{4} = 3024$.

54.(A) Let x_0 denotes the empty seats to the left of the first person, x_i (i = 1, 2, 3) be the number of empty seats between i th and $(i + 1)^{\text{th}}$ person and x_4 be the number of empty seats to the right of 4^{th} person.

Total number seats are 12. So, we can make this equation:

$x_0 + x_1 + x_2 + x_3 + x_4 = 8$ . . . **(i)**

Number of ways to give seats to 4 persons so that there should be two empty seats between any two persons is same as the number of integral solutions of the equation (i) subjected to the following conditions.

Conditions on x_1, x_2, x_3, x_4 :

According to the given condition, these should be two empty seats between any two persons. i.e.

Min $(x_i) = 2$ for $i = 1, 2, 3$ and Min $(x_0) = 0$

Max $(x_0) = 8 -$ Min $(x_1 + x_2 + x_3 + x_4) = 8 - (2 + 2 + 2 - 0) = 2$

Max $(x_4) = 8 -$ Min $(x_0 + x_1 + x_2 + x_3) = 8 - (2 + 2 + 2 - 0) = 2$

Similarly,

Max $(x_i) = 4$ for $i = 1, 2, 3$

No. of integral solutions of the equation (i) subjected to the above condition

= coeff of x^8 in the expansion of $(1 + x + x^2)^2 (x^2 + x^3 + x^4)^3$

= coeff of x^8 in $x^6 (1 + x + x^2)^5$= coeff off x^2 in $(1 - x^3)^5 (1 - x)^{-5}$

= coeff of x^2 in $(1 - x)^{-5} = {}^{5+2-1}C_2 = {}^6C_2 = 15.$

Number of ways to select 4 seats so that there should be atleast two empty seats between any two persons =15. But 4 persons can be arranged in 4 seats in 4! ways.

So total number of ways to arrange 4 persons in 12 seats according to the given condition = 15 × 4! = 360

55.(C) As every person should have exactly one neighbour, divide 4 persons into groups consisting two persons in each group.

Let G_1 and G_2 be the two groups in which 4 persons are divided.

According to the given condition G_1 and G_2 should be separated from each other.

Number of ways to select seats so that G_1 and G_2 are separated = ${}^{8+1}C_2$

But 4 persons can be arranged in 4 seats in 4! ways.

So total no. of ways to arrange 4 persons so that every person has exactly one neighbour = ${}^9C_2 \times 4! = 864$

56.(A) Considering CC as single object, U, CC, E can be arranged in 3! ways.

X U X C C X E X

Now the three S are to be placed in the 4 available places (X) so that C C are not separated but S are separated. No. of ways to place S S S = (No. of ways to select 3 places) × 1 = ${}^4C_3 \times 1 = 4$

$\Rightarrow$ No. of words = 3! × 4 = 24

57.(C) Let us first find the words in which no two S are together. To achieve this, we have to do following operations.

(i) Arrange the remaining letters U C C E in ways.

(ii) Place the three S S S in any arrangement from (i)

X U X C X C X E X

There are five available places for three S S S.

No. of placements = ${}^5C_3$

Hence total number of words with no two S together = $\frac{4!}{2!}\,{}^5C_3 = 120$.

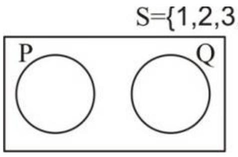
No. of words having C C separated and S S S separated = (No. of words having S S S separated) – (No. of words having S S S separated but C C together)

= 120 – 24 = 96 [using result of part (a)]

Miscellaneous	**Level - 2**

58.(B) $S = \{1, 2, 3, 4\}$

Let P and Q be two subsets of S such that $P \cap Q = \phi$



Now, each element of S has 3 choices $i.e.$, P or Q or $(P \cup Q)'$

$\therefore$ Number of ordered pairs of disjoint sets formed $= 3 \times 3 \times 3 \times 3 = 81$

59.(B) Let x_1, x_2, x_3 and x_4 be the marks obtained in papers 1, 2, 3, 4 respectively.

The total number of marks to be obtained by the candidate is $3n$.

Therefore, sum of marks obtained in various papers $= 3n$.

$\Rightarrow \quad x_1 + x_2 + x_3 + x_4 = 3n$ **.... (i)**

The total number of ways of getting $3n$ marks

= No. of solutions of the integral equation (i)

= Coefficient of x^{3n} in $\left(x^0 + x^1 + x^2 + ... + x^n\right)^3 \times \left(x^0 + x^1 + ... + x^{2n}\right)$

= coefficient of x^{3n} in $\left(\frac{1 - x^{n+1}}{1 - x}\right)^3 \left(\frac{1 - x^{2n+1}}{1 - x}\right)$

= coefficient of x^{3n} in $\left(1 - x^{n+1}\right)^3 \left(1 - x^{2n+1}\right)(1 - x)^{-4}$

= coefficient of x^{3n} in $\left[\left(1 - 3x^{n+1} + 3x^{2n+2} - x^{3n+3}\right)\left(1 - x^{2n+1}\right)(1 - x)^{-4}\right]$

= coefficient of x^{3n} in $\left[\left(1 - 3x^{n+1} - x^{2n+1} + 3x^{2n+2}\right)(1 - x)^{-4}\right]$

= coefficient of x^{3n} in $(1 - x)^{-4} - 3$ coefficient of x^{2n-1} in $(1 - x)^{-4}$ – coefficient of x^{n-1} in $(1 - x)^{-4} + 3$ coefficient of x^{n-2} in $(1 - x)^{-4}$

$= {}^{3n+4-1}C_{3n} - 3 \times {}^{2n-1+4-1}C_{2n-1} - {}^{n-1+4-1}C_{n-1} + 3 \times {}^{n-2+4-1}C_{n-2}$

$= {}^{3n+3}C_3 - 3\times {}^{2n+2}C_3 - {}^{n+2}C_3 + 3\times {}^{n+1}C_3 \qquad \left[as\ {}^nC_r = {}^nC_{n-r}\right]$

$= \frac{(3n+3)(3n+2)(3n+1)}{6} - 3\frac{(2n+2)(2n+1)(2n)}{6} - \frac{(n+2)(n+1)(n)}{6} + 3\frac{(n+1)(n)(n-1)}{6}$

$= \frac{1}{6}(n+1)(5n^2+10n+6)$

60.(B) Total no. of words $= \frac{9!}{2!}$

Number of words having "HIN" together = 7!; Number of words having "DUS" together = 7! / 2!

Number of words having "TAN" together = 7!; Number of words having "HIN" and "DUS" together = 5!

Number of words having "HIN" and TAN" together = 5!

Number of words having "DUS" and "TAN" together = 5!

Number of words having HIN, DUS and TAN together = 3!

Number of words required $\frac{9!}{3!} - \left(71 + \frac{7!}{2!} + 7!\right) + (5! + 5! + 5!) - 3! = 169194$

61.(C) The number of ways of placing all 6 letters in wrong envelopes

$= \lfloor 6\left[1 - \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} - \frac{1}{\lfloor 3} + \frac{1}{\lfloor 4} - \frac{1}{\lfloor 5} + \frac{1}{\lfloor 6}\right] = 265$

1 can go wrong to envelopes numbered 2, 3, 4, 5, 6 *i.e.*, 5 equally likely cases.

So, 1 will go to envelope numbered 2 in $\frac{265}{5} = 53$ ways.

62.(A) The number of subsets of A containing exactly three elements is ${}^nC_3$ whereas the number of three subsets of A that contain a_1, is ${}^{n-1}C_2$. We are given,

${}^{n-1}C_2 = \frac{20}{100}\left({}^nC_3\right) \Rightarrow \frac{(n-1)(n-2)}{2} = \frac{1}{5}\frac{n(n-1)(n-2)}{(6)} \qquad \Rightarrow \qquad n = 15$

63.(C) Each object can be put either in box B_1 (say) or in box B_2 (say). So, there are two choices for each of the n objects. Therefore, the number of choices for n distinct objects is $\underbrace{2\times 2\times ...\times 2}_{n-times} = 2^n$. In one-way B_1 is empty and in one other way B_2 is empty.

64.(B) When three digits are identical and the remaining two are also identical, then the number of ways $= \left({}^3C_1\right)\left({}^2C_1\right)\left(\frac{5!}{3!\,2!}\right) = 60$

Second case: Remaining two are different.

${}^3C_1 \times \frac{5!}{3!} = 60$ is the number of ways.

Thus, the number of such numbers = 120

65.(D) We have $\dfrac{a_n}{a_{n+1}} = \dfrac{10^n}{n!} \times \dfrac{(n+1)!}{10^{n+1}} = \dfrac{n+1}{10}$

Note that $\dfrac{a_1}{a_2} < 1, \dfrac{a_2}{a_3} < 1, \dfrac{a_3}{a_4} < 1, \dfrac{a_4}{a_5} < 1, \ldots \dfrac{a_8}{a_9} < 1, \dfrac{a_9}{a_{10}} = 1, \dfrac{a_{10}}{a_{11}} > 1, \dfrac{a_{11}}{a_{12}} > 1, \ldots$

$\therefore$ a_n is greatest if $n = 9$ or 10.

66.(B) Since 3 does not occur in 1000. So, we have to count the number of times 3 occurs when we list the integers from 1 to 999. Any number between 1 and 999 is of the form *abc* where $0 \le a, b, c \le 9$.

Clearly, Number of times 3 occurs = (No. of numbers in which 3 occurs exactly at one place) + 2 (No. of numbers in which 3 occurs exactly at two places) + 3(No. of numbers in which 3 occurs exactly at three places)

Now, No. of numbers in which 3 occurs exactly at one place: Since 3 can occur at one place in ${}^3C_1$ ways and each of the remaining two places can be filled in 9 ways. So, number of numbers in which 3 occurs exactly at one place = ${}^3C_1 \times 9 \times 9$.

No. of numbers in which 3 occurs exactly at two places: Since 3 can occur exactly at two places in ${}^3C_2$ ways and the remaining place can be filled in 9 ways.

So, number of numbers in which 3 occurs exactly at two places $= {}^3C_2 \times 9$.

No. of numbers in which 3 occurs at all the three places = 1. Since 3 can occur in all the three digits in one way only. So, number of numbers in which 3 occurs at all the three places is one.

Hence, Numbers of times 3 occurs $= {}^3C_1 \times 9 \times 9 + 2\left({}^3C_2 \times 9\right) + 3 \times 1 = 300$

67.(A) Common difference of the A.P. can be 1, 2, 3, ..., n. The number of AP's with 1, 2, 3, ..., n common differences are $(2n-1), (2n-3), \ldots, 1$ respectively

So, total number of AP's $= (2n-1) + \ldots + 1 = n^2$.

68.(D) Use unit's digit of 17^{2009} is same as that of the unit's digit of 7^{2009}

69.(C) Given, $n(A) = 2$ and $n(B) = 4$ $\Rightarrow$ $n(A \times B) = 8$

The number of subsets of A × B having 3 or more elements

$= {}^8C_3 + {}^8C_4 + \ldots + {}^8C_8 = 2^8 - {}^8C_0 - {}^8C_1 - {}^8C_2 = 256 - 1 - 8 - 28 = 219$

70.(C) Required number of ways $= {}^{12}C_4 \times {}^8C_4 \times {}^4C_4 = \dfrac{12!}{8! \times 4!} \times \dfrac{8!}{4! \times 4!} \times 1 = \dfrac{12!}{(4!)^3}$

71.(D) Since the balls are to be arranged in a row so that the adjacent balls are of different colours, therefore we can begin with a white ball or a black ball. If we begin with a white ball, we find that $(n+1)$ white balls numbered 1 to $(n+1)$ can be arranged in a row in $(n+1)!$ ways. Now $(n+2)$ places are created between $n+1$ white balls which can be filled by $(n+1)$ black balls in $(n+1)!$ ways.

So, the total number of arrangements in which adjacent balls are of different colours and first ball is a white ball is $(n+1)!\times(n+1)! = [(n+1)!]^2$. But we can begin with a black ball also. Hence the required number of arrangements is $2[(n+1)!]^2$.

72.(A) Corresponding to every four vertices, there's exactly one point of intersection, therefore total number of points of intersection is ${}^{2018}C_4$.

73.(D) Total number of parallelograms is equal to the total number of ways in which one can choose two parallel lines, from each set of "m + 2" parallel lines

$$= {}^{m+2}C_2 \cdot {}^{m+2}C_2 = \frac{(m+2)^2 (m+1)^2}{4}$$

74.(B) The "n" digit numbers will contain the digits 2, 3, 4, 5, 6, 7.

Using PIE, total number of such numbers $= 6^n - {}^2C_1 \cdot 5^n + {}^2C_2 \cdot 4^n$.

75.(ABCD) Correction in the question: $x, y, z \in \{1, 2, ...n\}$

Say z takes a value equal to "r" so x and y can take r values. Therefore, corresponding to one value of z, the total number of values (x, y) can take is r^2. Total number of such cases

$$= \sum_{r=1}^{n} r^2$$

Alternatively: 3 cases arise:

Only 1 number is selected: ${}^nC_1$ such triplets

Exactly 2 numbers are selected: ${}^nC_2 \times 3$ such triplets

[Only 1 choice for z and $(2^2 - 1)$ choices for (x, y)]

Exactly 3 numbers are selected: ${}^nC_3 \times 2$ such triplets.

[Only 1 choice for z and 2! Possible ways to determine (x, y)]

$${}^nC_1 + 3\,{}^nC_2 + 2\,{}^nC_3 = {}^{n+1}C_2 + 2\,{}^{n+1}C_3 = {}^{n+2}C_3 + {}^{n+1}C_3 \text{ and } 2\left[{}^{n+1}C_3\right] - {}^{n+1}C_2.$$

76.(AC) $3^p = (4-1)^p = 4\lambda_1 + (-1)^p$

$5^q = (4+1)^q = 4\lambda_2 + 1$

$7^r = (8-1)^r = 4\lambda_3 + (-1)^r$.

Hence any positive integer power of 5 will be in the form of $4\lambda_2 + 1$. Even power of 3 and 7 will be in the form of $4\lambda + 1$ and odd power of 3 and 7 will be in the form of $4\lambda - 1$. Hence, both p and r must be odd, or both must be even. Thus $p + r$ is always even.

Also, $p + q + r$ can be odd or even.

77.(ABD) Exponent of 2 is $\left[\frac{10}{2}\right] + \left[\frac{10}{2^2}\right] + \left[\frac{10}{2^3}\right] = 5 + 2 + 1 = 8$;

Exponent of 3 is $\left[\frac{10}{3}\right]+\left[\frac{10}{3^2}\right]=3+1=4$

Exponent of 5 is $\left[\frac{10}{5}\right]=2$ and Exponent of 7 is $\left[\frac{10}{7}\right]=1$

The number of divisors of 10! is (8+1) (4+1) (2+1) (1+1) = 270. The number of ways of putting *N* as a product of two natural numbers is 270/2 = 135.

78.(D) $^{12}C_4$ [Selection of any 4 points] - $\left[{}^5C_4+{}^4C_4\right]$ {All 4 points collinear}

$-\left[{}^5C_3\times{}^7C_1+{}^4C_3\times{}^8C_1\right]$ {Exactly 3 points collinear}

79.(ABC) By principle of homogeneity $x=y$ is evident and since sum of digits is either even or odd.

$\Rightarrow$ $x+y=$ total 5-digit numbers $=9\times10\times10\times10\times10$

$\Rightarrow$ $2x=90000$ $\Rightarrow$ $x=45000$

80.(ABC) When $n=3k$, there are exactly $n/3$ integers of each type $3p$, $3p+1$, $3p+2$. Now, sum of three selected integers is divisible by 3. Then either all the integers are of the same type $3p, 3p+1$ or $3p+2$ or one- one integer from each type. Then number of selection ways is

$${}^{n/3}C_3+{}^{n/3}C_3+{}^{n/3}C_3+\left({}^{n/3}C_1\right)^3=3\left({}^{n/3}C_3\right)+\left(n/3\right)^3$$

If $n=3k+1$, then there are $(n-1)/3$ integers of the type $3p$, $3p+2$ and $(n+2)/3$ integers of the type $3p+1$. Then number of selections is

$2\left({}^{(n-1)/3}C_3\right)+\left({}^{(n+2)/3}C_3\right)+\left((n-1)/3\right)^2(n+2)$. If $n=3k+2$, we can proceed similarly.

81.(ABCD) No. of permutations = $\frac{\lfloor 2n}{\lfloor n\,\lfloor n}={}^{2n}C_n$ (where *n* *a*'s and *n* *b*'s)

82.(C) (i) Miss C is taken

(A) B included $\Rightarrow$ A excluded $\Rightarrow$ ${}^4C_1\times{}^4C_2=24$

(B) B excluded $\Rightarrow$ ${}^4C_1\times{}^5C_3=40$

(ii) Miss C is not taken

$\Rightarrow$ B does not come $\Rightarrow$ ${}^4C_2\times{}^5C_3=60$ $\Rightarrow$ Total $=124$

Alternate Method:

Case I: Mr. B is present $\Rightarrow$ A is excluded, and C included

Hence, the number of ways is ${}^4C_2\,{}^4C_1=24$.

Case II: Mr. 'B' is absent $\Rightarrow$ No constraint

Hence, the number of ways is ${}^5C_3\,{}^5C_2=100$. $\therefore$ Total $=124$

83.(A) Any 4 points when selected gives one interior point of intersection

which can be done in ${}^nC_4$ ways ${}^nC_4=70$

$\Rightarrow$ $n(n-1)(n-2)(n-3) = 4! \times 70$

$\Rightarrow$ $n(n-1)(n-2)(n-3) = 8 \times 7 \times 6 \times 5$ $\Rightarrow$ $n = 8$ $\Rightarrow$ Diagonals = ${}^{n}C_2 - n = 20$

84.(150) **Method - 1**

The five balls can be distributed in 3 non-identical boxes in the following 2 ways

Boxes	**Box 1**	**Box 2**	**Box 3**
Number of balls	3	1	1
Number of balls	2	2	1

Case - I : 3 in one Box, 1 in another and 1 in third Box (3, 1, 1) **. . . (i)**

Number of ways to divide balls corresponding to (i)

$= \frac{5!}{3!1!1!} \frac{1}{2!} = 10$ [using 6.3 (c)]

But corresponding to each division there are 3! ways of distributing the balls into 3 boxes.

So, number of ways of distributing balls corresponding to (i)

= (No. of ways to divide balls) × 3! = 10 × 3! = 60

Case - II : 2 in one Box, 2 in another and 1 in third Box (2, 2, 1) **. . . (ii)**

Number of ways to divide balls corresponding to (ii)

$= \frac{5!}{2!2!1!} \frac{1}{2!} = 15$

But corresponding to each division there are 3! ways of distributing of balls into 3 boxes.

So, number of ways of distributing balls corresponding to (ii)

= (No. of ways to divide balls) × 3! = 15 × 3! = 90

Hence, required number of ways = 60 + 90 = 150

Method - 2

Using Result 6.4 (b),

Number of ways of distributing 5 balls in 3 Boxes so that no Box is empty

$r^n - {}^{r}C_1 (r-1)^n + {}^{r}C_2 (r-2)^n - {}^{r}C_2 (r-3)^n + \ldots\ldots$

Put $n = 5$ and $r = 3$ to get:

Number of ways = $3^5 - {}^{3}C_1\, 2^5 + {}^{3}C_1\, 1^5 = 243 - 3 \times 32 + 3 = 246 - 96 =$ 150 ways.

85. **(A)** $\frac{52!}{(13!)^4}$ **(B)** $\frac{52!}{4!(13!)^4}$

86.(C) Let the boys gets a, $a + b$ and $a + b + c$ toys respectively.

$a + (a + b) + (a + b + c) = 14$, $a \geq 1, b \geq 1 ; c \geq 1$

$\Rightarrow$ $3a + 2b + c = 14$, $a \geq 1, b \geq 1 ; c \geq 1$

$\therefore$ The number of solutions = Coefficient of t^{14} in $\{(t^3 + t^6 + t^9 + \ldots)(t^2 + t^4 + \ldots)(t + t^2 + \ldots)\}$

$\Rightarrow$ Coeff. of t^8 in $\{(1 + t^3 + t^6 + \ldots)(1 + t^2 + t^4 + \ldots)(1 + t + t^2 + \ldots)\}$

$\Rightarrow$ Coeff. of t^8 in $\{(1 + t^2 + t^3 + t^4 + t^5 + 2t^6 + t^7 + 2t^8)(1 + t + t^2 + \ldots + t^8)\}$

$= 1 + 1 + 1 + 1 + 1 + 2 + 1 + 2 = 10.$

Since, three distinct numbers can be assigned to three boys in 3! ways.

So, total number of ways = 10 × 3! = 60.

87.(B) Let x, y, z denote number of M, A, T coupons respectively.

Total selections is number of solutions of $x + y + z = k$ with $x, y, z \geq 0$ $= {}^{k+2}C_2$

Bad selections equal to ${}^{k-1}C_2$, $(x, y, z \geq 1)$

$\Rightarrow \quad {}^{k+2}C_2 - {}^{k-1}C_2 = 93 \qquad \Rightarrow \quad k = 31$

88.(C) $A \to m$ elements, $B \to n$ elements

No. of functions = $n \times n$ . . . . . . m times = n^m

As each element of set A has n options.

89.(AB) $x_1 + x_2 + x_3 \geq 210$

x_1 marks in P
marks in C x_2
marks in M x_3

No. of solutions of $x_1 + x_2 + x_3 + x_4 = 300$

$0 \leq x_i \leq 100,\ 0 \leq x_4 \leq 90$

$i = 1, 2, 3$

= coeff. of x^{300} in $(x^0 + x^1 + \ldots. x^{100})^3\ (x^0 + \ldots . x^{90})$

= coeff. of x^{300} in $\left[\dfrac{1 - x^{101}}{1 - x}\right]^3 \left[\dfrac{1 - x^{91}}{1 - x}\right]$

= coeff. of x^{300} in $({}^3C_0 - {}^3C_1\, x^{101} + {}^3C_2\, x^{202})\ (1 - x^{91})\ (1 - x)^{-4}$

= coeff. of x^{300} in $(1 - 3\,x^{101} + 3\,x^{202} - x^{91} + 3x^{192} - 3x^{293})\ (1 - x)^{-4}$

$$\begin{array}{cccccc} (1-x)^{-4} & -3(1-x)^{-4} & +3(1-x)^{-4} & -(1-x)^{-4} & +3(1-x)^{-4} & -3(1-x)^{-4} \\ \Downarrow & \Downarrow & \Downarrow & \Downarrow & \Downarrow & \Downarrow \\ x^{300} & x^{199} & x^{98} & x^{209} & x^{108} & x^{7} \end{array}$$

$= {}^{4+300-1}C_{300} - 3 \times {}^{4+199-1}C_{199} + 3\ {}^{4+98-1}C_{98} - {}^{4+209-1}C_{209} + 3\ {}^{4+108-1}C_{108} - 3\ {}^{4+7-1}C_7$

$= {}^{303}C_3 - 3\ {}^{202}C_3 + 3\ {}^{101}C_3 - {}^{212}C_3 + 3\ {}^{111}C_3 - 3\ {}^{10}C_3 = 129766$

Alternatively: Let $y_i = 100 - x_i$.

Now, $\sum y_i \leq 90$ with $0 \leq y_i \leq 100$. Number of solutions $= {}^{93}C_3$

90.(630) 7 diff. balls in 3 diff. boxes

B - 1	**B - 2**	**B - 3**	**ways**
3	2	2	Section 6.3

$\dfrac{7!}{3!\,2!\,2!} \times \dfrac{1}{2!} \times 3!$

Total ways = 630

91.(ABC) n objects are already arranged in a row.

3 objects are to be selected such that no two of them are next to each other.

We will think it as objects are already placed and we have to place between them 3 objects such that no two of them are next to each other which can be done in ${}^{n-3+1}C_3$ ways. = ${}^{n-2}C_3$

[You must have seen a (TPC) of placing the restricted items such that they are always separated].

Points to be noted :

(i) First arrange those which are without restriction.

(ii) Find the no. of places available

Ex : | A | B | C | , 4 places (|) are available.

Hence it can be done in ${}^{n-2}C_3$ ways $\equiv \frac{(n-2)(n-3)(n-4)}{6}$

Also, ${}^{n-3}C_2 + {}^{n-3}C_3 = {}^{n-2}C_3$

92. ${}^nC_3 - {}^nC_1 \cdot (n-2) + n$

A n-sided polygon has n angular points. Number of triangles formed from these n angular points = ${}^nC_3$.

But it also includes the triangles with sides on the polygon.

Let us consider a side PQ. If each angular point of the remaining (n – 2) points is joined with PQ, we get a triangle with one side PQ.

$\therefore$ Number of triangles with PQ as one side

= n –2 In similar ways n sides like QR can be considered. Hence number of triangle

= n (n –2).

But some triangles have been counted twice. For example, PQ side with R gives ΔPQR and QR side with P gives same ΔPQR.

Number of such triangles = n

[$\therefore$ For each side, one triangle is repeated. Hence for n sides, $n \Delta$ s has been counted more.]

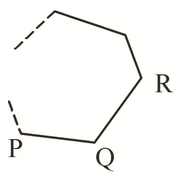
Hence, the number of triangles of which one side is the side of the triangle.

= n (n – 2) – n = n (n–3)

Hence number of required triangles

= ${}^nC_3 - n\,(n-3)$

$= \frac{n(n-1)(n-2)}{6} - n(n-3)$



$= \frac{n}{6}(n^2 - 9n + 20)$

$= \frac{n}{6}(n-4)(n-5)$

93.(121) When coefficient occurs in these kinds of questions, use counting method.

S. No.	x	$y + z$	No. of solutions
1	0	20	21
2	1	18	19
3	2	16	17
|	|	|	|
|	|	|	|
|	|	|	|
|	|	|	|
|	|	|	|
|	|	|	|
|	|	|	|
21	20	0	1

No. of solutions = 1 + 3 + ...+ 21 = 121.

94. **(i)** $x + y + z + w = 20$; $x \geq 0$, $y \geq 0$, $z \geq 0$, $w \geq 0$

Coeff. of α^{20} in $(\alpha^0 + \alpha^1 + \alpha^2 + ...)^4$

$= (1 - \alpha)^{-4} = {}^{20+4-1}C_{20} = {}^{23}C_3 = 1771$

Note: You can directly use the result ${}^{n+r-1}C_{r-1}$

(ii) Number of ways = coeff. of α^{20} in $(\alpha + \alpha^2 + \alpha^3 + \ldots\ldots \infty)^4$

= coeff. of α^{20} in $\alpha^4 (1-\alpha)^{-4}$ or = coefficient of α^{16} in $(1-\alpha)^{-4} = {}^{19}C_{16} = 969$

Note: that you can directly use ${}^{n-1}C_{r-1}$

(iii) Number of solutions = Coefficient of α^{20} in $(\alpha + \ldots \alpha^{10})^4$

= coefficient of α^{16} in $(1 + \alpha \ldots \alpha^9)^4 = (1-\alpha)^{-4} (1-\alpha^{10})^4 = {}^{19}C_3 - {}^4C_1 \times {}^9C_3$

(iv) Each variable is an odd number.

$\therefore$ $x = 2x_1 + 1$ $y = 2y_1 + 1$

$z = 2z_1 + 1$ $w = 2w_1 + 1$ [where $x_1, y_1, z_1, w_1 \geq 0$]

$x + y + z + w = 20$

$\Rightarrow (2x_1 + 1) + (2y_1 + 1) + (2z_1 + 1) + (2w_1 + 1) = 20$

$2x_1 + 2y_1 + 2z_1 + 2w_1 = 16$

$\Rightarrow x_1 + y_1 + z_1 + w_1 = 8$ [where $x_1, y_1, z_1, w_1 \geq 0$]

Number of solutions = ${}^{8+4-1}C_{4-1} = {}^{11}C_3 = 165$

(v) Assume $0 < x < y < z < w$ (4! Possible permutations)

Let $x = x_1$

$y = x + x_2 = (x_1) + x_2$

$z = y + x_3 = (x_1 + x_2) + x_3$

$w = z + x_4 = (x_1 + x_2 + x_3) + x_4$ [where $x_1, x_2, x_3, \geq 1$]

$x + y + z + 2 = 20$

$\Rightarrow x_1 + (x_1 + x_2) + (x_1 + x_2 + x_3) + (x_1 + x_2 + x_3 + x_4) = 20$

$4x_1 + 3x_2 + 2x_3 + x_4 = 20$ . . . **(i)** [where $x_1, x_2, x_3, x_4 \geq 1$]

Let us again change the variables

$x_1 = y_1 + 1$; $x_2 = y_2 + 1$; $x_3 = y_3 + 1$; $x_4 = y_4 + 1$ [where $y_1, y_2, y_3, y_4 \geq 0$]

Substituting above values in (i) $4(y_1 + 1) + 3(y_2 + 1) + 2(y_3 + 1) + (y_4 + 1) = 20$.

$\Rightarrow 4y_1 + 3y_2 + 2y_3 + y_4 = 10$ [where $y_1, y_2, y_3, y_4 \geq 0$]

y_1	$3y_2 + 2y_3 + y_4$	No. of solutions
0	10	14 (Use Table –1)
1	6	7 (Use Table –2)
2	2	2 (Use Table –2)

Table -1

$3y_2 + 2y_3 + y_4 = 10$

y_2	$2y_3 + y_4$	No. of solutions
0	10	6
1	7	4
2	4	3
3	1	1
		14

Table -2

$3y_2 + 2y_3 + y_4 = 6$

y_2	$2y_3 + y_4$	No. of solutions
0	6	4
1	3	2
2	0	1
		7

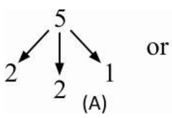
Table - 3

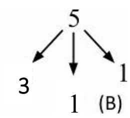
$3y_2 + 2y_3 + y_4 = 2$

y_2	$2y_3 + y_4$	No. of solutions
0	2	2
		2

Total solutions = 23 × 24

95.(B) (P) Divide 5 identical balls in 3 diff. boxes such that each gets atleast one = $^{5-1}C_{3-1} = 6$.

(Q) 

or 

(A): $\dfrac{\lfloor 5}{\lfloor 2 \lfloor 2 \lfloor 1} \times \dfrac{1}{\lfloor 2}$ **(B):** $\dfrac{\lfloor 5}{\lfloor 3 \lfloor 2 \lfloor 1} \times \dfrac{1}{\lfloor 2}$ $= 25$

(R)

B	B	B
3 2	1 2	1 1

Number of ways = 2

(S) Divide 5 diff. balls in 3 diff. boxes such that boxes can remain empty = $3^5 = 243$